Honors Algebra 2

Composite Functions

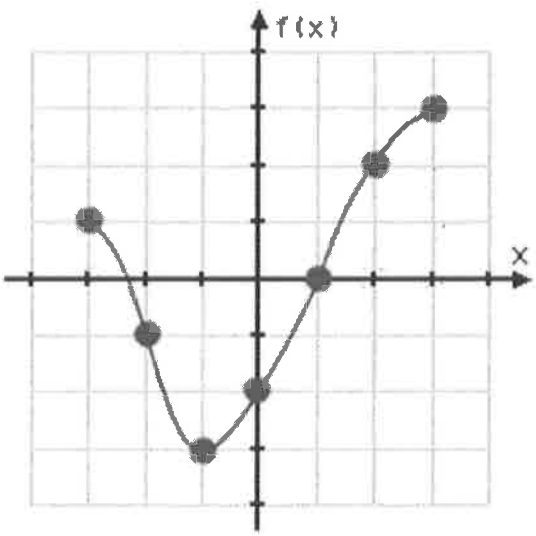
1. Given f(x) and g(x) find the following:

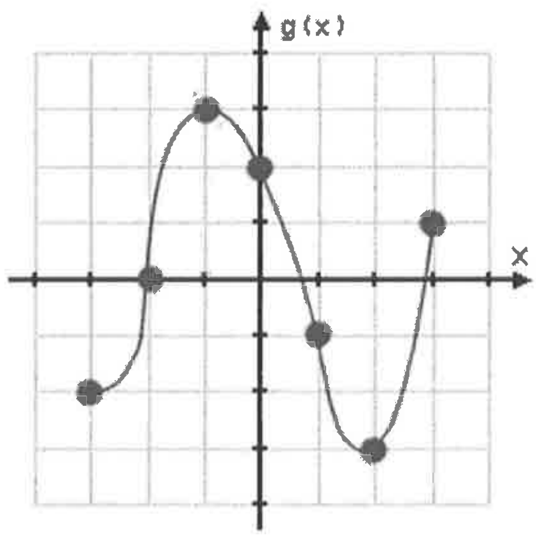
f(x) = (-2,3)(-1,1) (0,0)(1, -1) (2, -3)

g(x)= (-3, 1) (-1, -2)(0, 2)(2, 2)(3, 1)

a. f(1)   b. g(0)   c. g(f(1))

d. f(g(2))   e. f(f(1))   f. g(f(0))





2. Given the graphs of f(x) and g(x) above, find the following:

a. f(g(-1)) f(3)
=3

b. g(f(-3))
g(1) = -1

c. f(g(0))
f(2) = 2

3. Show the two composite functions are equal

$$f(x) = 2x - 6 \; and \; g(x) = \frac{1}{2}(x+6)$$

Prove that f(g(x))= g(f(x))=x

$2(\frac{1}{2}(x+6))-6$ $\qquad$ $\frac{1}{2}(2x-6+6)$

$x+6-6$ $\qquad$ $\frac{1}{2}(2x)$

$\boxed{x \quad = \quad x}$

4. Given $f(x) = \frac{x-6}{x}$ $and\; g(x) = x^2 + 9, find\; (g°f)(-2)$

$g(f(-2))$ $\qquad$ $f(-2) = \frac{-2-6}{-2} = \frac{-8}{-2} = 4$

$g(4) = 4^2 + 9$

$= 16 + 9$

$\boxed{= 25}$

Find the composite function and domain of each.

5. $(f°g)(x): f(x) = 4x + 12, g(x) = 5x - 1$

$f(g(x)) = 4(5x-1) + 12$

$= 20x - 4 + 12$

$\boxed{= 20x + 8 \qquad D: \mathbb{R}}$

6. $(g°f)(x): f(x) = -2x + 3, g(x) = 6x + 4$

$g(f(x)) = 6(-2x+3) + 4$

$= -12x + 18 + 4$

$\boxed{= -12x + 22 \qquad D: \mathbb{R}}$

7. $(f°g)(x): f(x) = \frac{1}{x+5}, g(x) = \frac{7}{8x}$

$f(g(x)) = \dfrac{1}{\frac{7}{8x} + \frac{5(8x)}{8x}} = \dfrac{1}{\frac{40x+7}{8x}}$ $\boxed{= \dfrac{8x}{40x+7} \qquad D: x \neq \frac{-5}{0}, -7/40}$

8. $(g°f)(x): f(x) = \frac{x-7}{3}, g(x) = 3x + 7$

$g(f(x)) = 3\left(\frac{x-7}{3}\right) + 7$

$\boxed{= x}$ D: $\mathbb{R}$

9. $(f°g)(x): f(x) = \sqrt{x+4}, g(x) = 8x - 8$

$f(g(x)) = \sqrt{8x-8+4}$

$\boxed{= \sqrt{8x-4} \quad D: x > \frac{1}{2}}$

10. $(g°f)(x): f(x) = 4x^2 + 3x + 5, g(x) = 3x - 8$

$g(f(x)) = 3(4x^2+3x+5) - 8$

$\cancel{12x^2+9x+15-8}$

$\boxed{= 12x^2+9x+7 \quad D: \mathbb{R}}$

Determine whether the composite functions are equal to x

11. $f(x) = \sqrt[5]{x-8}, g(x) = x^5 + 8$

$f(g(x))$

$\sqrt[5]{x^5+8-8}$

$\sqrt[5]{x^5}$

$\boxed{x = x}$

$g(f(x))$

$(\sqrt[5]{x-8})^5 + 8$

$x - 8 + 8$

x

X

12. $f(x) = x^2 + 5, g(x) = \sqrt{x} - 5$

$f(g(x))$

$(\sqrt{x}-5)^2 + 5$

$x - 10\sqrt{x} + 25$

$\boxed{\neq x}$

13. $f(x) = \sqrt{x+1}, g(x) = x^2$

$f(g(x))$

$= \sqrt{x^2+1}$

$g(f(x))$

$(\sqrt{x+1})^2$

$= x + 1$

$\boxed{\neq x}$

14. $f(x) = x^3 + 3, g(x) = \sqrt[3]{x-3}$

$f(g(x))$

$(\sqrt[3]{x-3})^3 + 3$

$x - 3 + 3$

$= x$

$\boxed{x = x}$

$g(f(x))$

$\sqrt[3]{x^3+3-3}$

$\sqrt[3]{x^3}$

$= x$

Find the functions f and g so that the composition of f and g is H

15. $H(x) = \sqrt[3]{x+1}$

$f(x) = \sqrt[3]{x}$ $g(x) = x+1$

16. $H(x) = (5 - 2x^3)^2$

$f(x) = x^2$ $g(x) = 5 - 2x^3$

17. $H(x) = \frac{1}{x^2-7}$

$f(x) = \frac{1}{x}$ $g(x) = x^2 - 7$

18. $H(x) = \frac{8}{\sqrt{9x+4}}$

$f(x) = \frac{8}{x}$ $g(x) = \sqrt{9x+4}$

19. $H(x) = |4 - 3x^2|$

$f(x) = |x|$ $g(x) = 4 - 3x^2$

20. $H(x) = |5x + 1|$

$f(x) = |x|$ $g(x) = 5x + 1$

For problems 21-26 find f °g, g°f, f°f and g°g for each pair of functions. State the domain of each composite function.

21. $f(x) = 2 - x, g(x) = 3x + 1$ D : R

f(g(x))= $1-3x$ g(f(x))= $7-3x$

f(f(x))= x g(g(x))= $9x+4$

22. $f(x) = 2x - 1, g(x) = 2x + 1$ D : R

f(g(x))= $4x+1$ g(f(x))= $4x-1$

f(f(x))= $4x-3$ g(g(x))= $4x+3$

23. $f(x) = 3x^2 + x + 1, g(x) = |3x|$ D : R

f(g(x))= $27x^2 + |3x| + 1$ g(f(x))= $|9x^2+3x+3|$

f(f(x))= $27x^4+18x^3+24x^2+7x+5$ g(g(x))= $9x$

24. $f(x) = \sqrt{3x}, g(x) = 1 + x + x^2$ $x \geq 0$

f(g(x))= $\sqrt{3x^2+3x+3}$ g(f(x))= $3x + \sqrt{3x} + 1$

f(f(x))= $\sqrt{3\sqrt{3x}}$ g(g(x))= $x^4+2x^3+4x^2+3x+3$

25. $f(x) = \frac{x+1}{x-1}, g(x) = \frac{1}{x}$ D: $x \neq 1, 0, -1$

f(g(x))= $\frac{1+x}{1-x}$ g(f(x))= $\frac{x-1}{x+1}$

f(f(x))= x g(g(x))= x

26. $f(x) = \sqrt{x-3}, g(x) = \frac{3}{x}$ D: $x > 0$

f(g(x))= $\sqrt{\frac{3-3x}{x}}$ g(f(x))= $\frac{3}{\sqrt{x-3}}$ $x > 3$

f(f(x))= $\sqrt{\sqrt{x-3}-3}$ g(g(x))= x $x \neq 0$